Summer School of the ANR-DFG project Effective Approximation and Dynamics of Many-Body Quantum Systems

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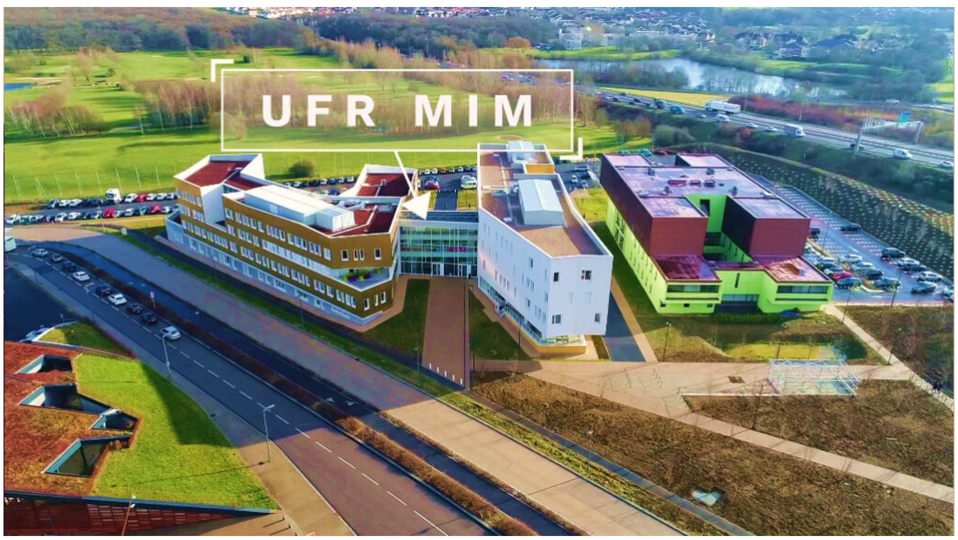
Metz, June 23-June 27, 2024

Institut Élie Cartan, UFR MIM - Mathématiques, Informatique et Mécanique,
Université de Lorraine, Metz, France



Access

France – Metz – Technopôle – 3 rue Augustin Fresnel – UFR MIM – Petit Amphithéâtre

By car Metz, 3 rue Augustin Fresnel
GPS : 49°05’41.5”N 6°13’47.2”E

By public transportation from either place de la république or Metz-Ville Train Station:
Bus: Mettis B – Direction: Hôpital Mercy – Stop: Grandes Écoles

Program

Monday	Tuesday	Wednesday	Thursday
9:00 – 10:00 M. Ballesteros	9:00 – 10:00 M. Ballesteros	9:00 – 10:00 M. Ballesteros	9:00 – 10:00 M. Falconi
10:30 – 11:30 M. Grillakis	10:30 – 11:30 M. Grillakis	10:30 – 11:30 M. Grillakis	10:30 – 11:30 J. Fröhlich
11:30 – 12:30 S. Cenatiempo	11:30 – 12:30 S. Cenatiempo	11:30 – 12:30 S. Cenatiempo	11:30 – 12:30 I. M. Sigal
14:30 – 15:30 M. Falconi	14:30 – 15:30 M. Falconi	Boat trip: Moselle river	
15:30 – 16:30 I. M. Sigal	15:30 – 16:30 I. M. Sigal	& hike: Saint Quentin Mount	
17:00 – 18:00 J. Fröhlich	Posters		

Social Events

Welcome diner
Cold buffet Sunday between 19:00 and 21:00 at Metz train station in the room “Salon de l’empereur”

Boat on the Moselle river & Hiking Wednesday at 14:55
Meeting point at the boat stop Ponton Metz’O Moyen Pont (*Below* rue de la garde)

Conference diner Wednesday evening at 20:00
Brasserie des Arts et Métiers: 2 Bis Rue Gambetta, 57000 Metz

Mini-Courses

MIGUEL BALLESTEROS — Universidad Nacional Autónoma de México

Mathematical aspects of non-relativistic quantum electrodynamics

In this mini-course we study the so-called non relativistic quantum electrodynamics (NR QED). NR QED describes non relativistic particles coupled to the quantized electromagnetic field and it is a low energy approximation of quantum electrodynamics. The course is intended to be accessible to students that have no background on the mathematical formalism of quantum field theory. For this reason, basic mathematical tools will be presented. We will cover, for example, operators defined on Fock spaces (including creation and annihilation operators) and we will state and prove some basic properties and results. In the last part of the course, we will present more advanced topics such as resonances.

SERENA CENATIEMPO — Gran Sasso Science Institute

Quantum Mechanics at our scale: a mathematical challenge

While the theory of quantum mechanics describes interactions between constituents of matter at the microscopic scale, the effects of these interactions may lead to fascinating quantum mechanics phenomena at a macroscopic scale. The discovery of such phenomena, including superfluidity and superconductivity as two pioneering examples, has led to extraordinary developments in realizing new materials that exploit the principles of quantum mechanics to exhibit innovative behaviours. In these lectures, we will discuss the challenge of developing mathematical models that rigorously describe how these macroscopic effects emerge from the microscopic scale, focusing on the paradigmatic example of the interacting Bose gas.

MARCO FALCONI — Politecnico di Milano

Scaling limits in quantum models of particle-field interaction

In these lectures I will review a series of recent results on the interaction between quantum particles and fields, in different scaling regimes (all somewhat related to Bohr's correspondence principle). From a mathematical perspective, infinite dimensional semiclassical analysis lies at the backbone of our results; we will review its main developments and connect them with the study of systems of quantum particles in interaction with a semiclassical field, as well as with the equations of classical electrodynamics.

JÜRG FRÖHLICH ——————— Eidgenössische Technische Hochschule Zürich

1) Why text-book Quantum Mechanics calls for a completion, and why the "ETH - Approach to Quantum Mechanics" may provide an adequate one.

2) Infinitesimal perturbation theory and quantum-Poisson processes

as quantum-mechanical descriptions of the stochastic evolution of the states of individual isolated systems.

MANOUSSOS GRILLAKIS ——————— University of Maryland

On the Evolution of interacting Bosons

I will consider the time evolution of N interacting Bosons (two body inter- action) via a potential of the form $N^{3\beta}V(|x-y|)$ where $0 < \beta \leq 1$. I will derive what is called the Hartree-Fock-Bogoliubov approximation which results in a coupled system of Schrödinger type equations. These equations describe the coupling between the mean field (Nonlinear cubic Schrödinger equation) with the wave function of a pair of particles.
Next I will explain some of the analysis needed to establish global in time existence of the limit equation (as $N \to \infty$) of the coupled system. This part involves establishing Strichartz type estimates independent of N where N is the number of particles, since we are interested in the limit $N \to \infty$ of the equations.
This work is in collaboration with Matei Machedon, Zehua Zhang, Jacky Chong as well as some recent seminal work of X. Huang.

Partial Differential Equations of Quantum Physics

1. Gross-Pitaevski equation (GPE)
The Gross-Pitaevski equation describes superfluids and Bose-Einstein condensates (Nobel prizes 1996 and 2001) in "macroscopic" regime. Exactly the same equation appeared in theories of plasma and surface waves where it is called the nonlinear Schrödinger equation (NLS). Mathematically, it is a key nonlinear Hamiltonian partial differential equation (PDE) and it has led to rich mathematics. In this lecture, I will give an introduction into the theory of the GPE, proceeding quickly to recent results and exciting open problems.

2. Ginzburg-Landau equations (GLE)
The Ginzburg-Landau equations (Nobel prize 2003) is perhaps the most successful PDE system in condensed matter physics. Originating in the theory superconductivity (Nobel prizes 1972 and 2003), it was extended to various other areas of condensed matter physics, such as theories of liquid crystals and fractional Hall effect (Nobel prizes 1991 and 1998). Independently, these equations appeared in particle physics providing the simplest U(1)-gauge theory. Their non-abelian generalization, the Yang-Mills-Higgs equations (Nobel prizes 1999, 2013) is a fundamental ingredient of the standard model of particle physics. GLE have also a natural geometrical interpretation coupling the section and connections of line bundles. In this lecture, I describe the basic properties of the GLE, its key solutions (magnetic vortices and vortex lattices), discuss recent results and say a few words on the extension of this equation to Riemann surfaces.

3. Kohn-Sham equation (density functional theory or DFT)
The Kohn-Sham equation (KSE) is a key equation of density functional theory (Nobel prize 1998), providing effective description of electronic structure of matter. Nowadays, most of the computations in quantum physics and quantum chemistry involving electrons is done using this equation. The mathematical theory for the KSE is presently under construction. In this lecture, I will describe the KSE, its main properties and the key existing results, outline some approaches and present open problems.

Ground States for Mass Critical Two Coupled Semi-Relativistic Hartree Equations with Attractive Interactions

Thi Anh Thu Doan *(Ton Duc Thang University)*

We prove the existence and nonexistence of $L^2(\mathbb{R}^3)$-normalized solutions of two coupled semi-relativistic Hartree equations, which arisen from the studies of boson stars and multi-component Bose–Einstein condensates. Under certain condition on the strength of intra-specie and inter-specie interactions, by proving some delicate energy estimates, we give a precise description on the concentration behavior of ground state solutions of the system. Furthermore, an optimal blowing up rate for the ground state solutions of the system is also proved.

Caldeira-Leggett model: Quasi-classical limit

*Michele Correggi, Marco Falconi, **Michele Fantechi** (Politecnico di Milano)*

The Caldeira-Leggett model is widely used to understand quantum decoherence and dissipation in quantum open systems. The model was originally treated by functional integration techniques or by deriving quantum Master equations. We propose to explore the same questions in a different regime in which the bosonic field coupled with the particle is highly excited, and can be treated as classical. We obtain an effective quasi-classical model for the small system by exploiting results on the semiclassical limit of bosonic fields.

Almost sure existence of global solution to an abstract initial value problem and application to PDEs and Quantum Mechanics

Shahnaz Farhat *(Constructor University, Bremen, Germany)*

This poster talks about the almost sure existence of global solutions for initial value problems of the form $\partial_t u = v(t,u)$ on separable dual Banach spaces. We prove a general result stating that whenever there exists $(\mu_t)_{t\in\mathbb{R}}$ a family of probability measures satisfying a related statistical Liouville equation, there exist global solutions to the initial value problem for μ_0-almost all initial data, possibly without uniqueness. The main assumption is a mild integrability condition of the vector field v with respect to $(\mu_t)_{t\in\mathbb{R}}$. Then, we apply similar relaxed statistical result to study the dynamics of relativistic or non-relativistic charged particles in interaction with a scalar field. Our main contribution is the derivation of the classical dynamics of a particle-field system as an effective equation of the quantum microscopic Nelson model, in the classical limit where the value of the Planck constant approaches zero ($\hbar \to 0$).

Spectral and scattering theory for some non-self-adjoint operators

Nicolas Frantz *(Paris Cergy Université)*

This poster outlines a work in colaboration with J. Faupin about a non-self-adjoint hamiltonian acting on a complex separable Hilbert space. It present a spectral decomposition formula and the study of the asymptotic of the solutions to the Schrödinger equation for this hamiltonian. These results apply to complex values Schrödinger operators with potentials which decays at infinity.

Semiclassical limit of entropies in the Gibbs state and Wehrl free energy

Raphaël Gautier *(Ecole Normale Supérieure Paris-Saclay École normale supérieure de Cachan - ENS Cachan)*

Entropy and Free energy are central concepts in both statistical physics and information theory with quantum and classical facets. In this work, we study the von Neumann and Wehrl entropies from the point of view of semi-classical analysis. In the first part, we prove the convergence of the von Neumann and Wehrl entropy for quantum Gibbs states (thermal equilibrium) in finite dimensional Hilbert space, after a suitable renormalization. In a second part, we prove the Gamma convergence of the free energies functionals associated to von Neumann and Wehrl entropy, and in particular the convergence of their minimums, and their minimizers are related to the Gibbs states. The main ingredients are the canonical commutation relations and their representation on the Fock space. In particular, we will use coherent states, the Wick and anti-Wick quantization and semiclassical measures.

Number of bound states for fractional Schrödinger operators

*Sébastien Breteaux, Jérémy Faupin, **Viviana Grasselli** (Université de Lorraine)*

We study the number of negative eigenvalues, counting multiplicities, of the fractional Schrödinger operator $H_s = (-\Delta)^s - V(x)$ on $L^2(\mathbb{R}^d)$, for any $d \geq 1$ and $s \geq d/2$. Our proof relies on a splitting of the Birman-Schwinger operator associated to this spectral problem into low- and high-energies parts, a projection of the low-energies part onto a suitable subspace, and, in the critical case $s = d/2$, a Cwikel-type estimate to handle the high-energies part.

The Majorant Method for the Fermionic Effective Action

***Wilhelm Kroschinsky** (Mathematisches Institut der Universität Bonn), Domingos H. U. Marchetti (University of São Paulo), Manfred Salmhofer (Heidelberg University)*

We revisit the problem of controlling Polchinski's equation by the solution of an associate Hamilton-Jacobi equation which determines a norm majorant for the fermionic effective action. This method, referred to as the Majorant Method, was first introduced by D. Brydges and J. Wright in 1988, but its original formulation contains a gap which has never been addressed. We overcome this gap and show that the majorant equation and its existence condition are analogous to the ones originally obtained by Brydges and Wright. As an application of the method, we investigate systems with quartic perturbations

Unique Continuation for Fourth Order Schrödinger Equations

***Zachary Lee** (University of Texas at Austin), Xueying Yu (Oregon State University)*

We study uniqueness properties of solutions to linear and non-linear fourth-order Schrödinger equations in any number of dimensions. We show that a soluion to the linear equation with fast decay in certain Sobolev spaces at two different times must vanish identically. We also show that if the difference between two solutions to the non-linear equation obeys the same fast decay property, the two solutions must coincide identically.

Momentum Distribution of a Fermi Gas in the Random Phase Approximation

*Niels Benedikter, **Sascha Lill** (Università degli Studi di Milano)*

Brockett-Wegner Flow: From Linear Algebra to Many-Body Quantum Fields Theory

***Nathan Metraud** (University of the Basque Country), Jean-Bernard Bru (University of the Basque Country, BCAM)*

Introduced in the nineties, the "double commutators" flow was initially meant to diagonalize symmetric matrices and solve linear programing problems. In infinite dimensions, highly non-trivial issues occur without a general solution. We present how we can use this flow in a specific case in many body quantum fields theory. We investigate the diagonalization of quadratic fermionic Hamiltonians, following the approach of Bach and Bru in the bosonic case, while highligthing the difference in the behavior of the flow due to the difference between the CAR and CCR that leads to either an elliptic or a hyperbolic flow.

On the Bogolubov-Hartree-Fock Energy of the Pauli-Fierz Hamiltonian

***Merten Mlinarzik** (Technische Universität Braunschweig)*

Following recent analyses of the Pauli-Fierz model, we investigate the corresponding Bogolubov-Hartree-Fock variational problem. Bach, Breteaux, and Tzaneteas (2013) showed that the minimum of the energy over quasifree states coincides with the minimum of the energy over pure quasifree states. Starting from this fact, we utilize techniques of Bach and Hach (2022) to study positivity properties of the minimizing Bogolubov transformation. Further symmetries of the minimizer are investigated with the aim of simplifying the energy functional. Finally, we modify a parametrization introduced by Bach and Hach to write the simplified energy functional as a sum of polynomials and resolvents of the variables.

Joint work with V. Bach and M. Herdzik.

2D rotating Bose-Einstein condensations at the critical speed

*Van Duong Dinh (École normale supérieure de Lyon CNRS-ENS Lyon), **Dinh Thi Nguyen** (Uppsala University), Nicolas Rougerie (École normale supérieure de Lyon CNRS-ENS Lyon)*

We study the minimizers of a magnetic 2D nonlinear Schrödinger energy functional in a quadratic trapping potential, describing a rotating Bose-Einstein condensate. In the first part, we consider the case of a repulsive interaction potential. We derive an effective Thomas-Fermi-like model in the rapidly rotating limit where the centrifugal force compensates the confinement. The available states are restricted to the lowest Landau level. The coupling constant of the Thomas-Fermi functional is to link the emergence of vortex lattices (the Abrikosov problem). In the second part, we consider the case an attractive interaction potential. When the strength of the interaction approaches a critical value, the system collapses to a profile obtained from the (unique) optimizer of a Gagliardo-Nirenberg interpolation inequality. This was established before in the case of fixed rotation frequency. We extend the result to rotation frequencies approaching, or even equal to, the critical frequency at which the centrifugal force compensates the trap. We prove that the blow-up scenario is to leading order unaffected by such a strong deconfinement mechanism. In particular the blow-up profile remains independent of the rotation frequency.

Weyl laws for interacting particles

***Ngoc Nhi Nguyen** (University of Milan)*

The known Weyl laws of Schrödinger operators provide asymptotics to the ground state density of systems of several non-interacting fermions submitted to an external potential. This poster presents the corresponding Weyl laws in the case of interactions between these particles.

Gyrokinetic limit of the 2D Hartree equation in a large magnetic field

***Denis Périce** (Constructor University)*

We study the dynamics of two-dimensional interacting fermions submitted to a homogeneous transverse magnetic field. We consider a large magnetic field regime, with the gap between Landau levels set to the same order as that of potential energy contributions. Within the mean-field approximation, i.e. starting from Hartree's equation for the first reduced density matrix, we derive a drift equation for the particle density. We use vortex coherent states and the associated Husimi function to define a semi-classical density almost satisfying the limiting equation. We then deduce convergence of the density of the true Hartree solution by a Dobrushin-type stability estimate for the limiting equation.

From Short-range to Mean-field Models in Quantum Lattices

***Kaue Rodrigues** (Basque Center for Applied Mathematics, Universidad del Pais Vasco / Euskal Herriko Unibertsitatea)*

In theoretical physics, mean-field models - such as the BCS model of superconductivity - have been extremely handy to prove analytical results that capture important real physical phenomena. However, it is not clear why they are a good approximation for realistic interactions, since effective interparticle interactions are widely seen as beig short-ranged (e.g., the Hubbard model, or Yukawa-type potentials). One way to establish a connection between short-range and mean-field models is via the so-called Kac limit. In this work we establish a precise mathematical relation between such limit and the thermodynamics of mean-field models with attractive, repulsive and mixed interactions, as well as the convergence of equilibrium states.

Microscopic derivation of Gibbs measures for the 1D focusing quintic NLS

Andrew Rout *(University of Rennes Université de Rennes I)*

We obtain a microscopic derivation of Gibbs measures for the quintic focusing nonlinear Schrödinger equation (NLS) on the torus from many-body quantum Gibbs states. We prove results in both the time-independent and time-dependent settings. This corresponds to a three-body interaction on the many-body level. This is a continuation of earlier work deriving cubic measures in the same setting, which corresponded to a two-body interaction in the many-body setting. This is the first such known result in the three-body regime.

To derive a well-defined measure, we truncate in the mass of the classical free field and the rescaled particle number in the quantum setting. Our methods are based on the perturbative expansion previously developed by Fröhlich, Knowles, Schlein, and Sohinger. This is based on work with Vedran Sohinger.

Derivation of the effective dynamics for the Bose Polaron

Jonas Lampart (Université de Bourgogne Laboratoire Interdisciplinaire Carnot de Bourgogne (ICB), UMR 6303 CNRS - Université de Bourgogne Franche-Comté), Peter Pickl (University of Tübingen), ***Siegfried Spruck*** *(University of Tübingen, Université de Bourgogne Laboratoire Interdisciplinaire Carnot de Bourgogne (ICB), UMR 6303 CNRS - Université de Bourgogne Franche-Comté)*

We study the dynamics of a dense Bose gas evolving in $\mathbb{R}^3$ in the presence of an impurity particle. The interaction of the Bose gas with the impurity is generally believed to lead to the formation of a quasi-particle called the Bose polaron. In the considered setting the Bose gas has high initial density ρ with large initial volume Λ. In this setting the Bose gas, without impurity, exhibits Bose-Einstein condensation with a few excitiations out of the ideal condensate. We prove the validity of an effective description of the dynamics by a model of particles interacting with the excitation field of Bogoliubov-type in a condensate of non-constant density. The effective Bogoliubov-Fröhlich Hamiltonian H^{BF} introduced this way describes the dynamics of the polaron and couples the impurity linearly to the field of excitations.

Participants

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Mathématiques et Sciences de l'Architecture de l'Université de Lorraine

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Programme ORION

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